

Uppgift 12.3

Deluppgift A

$$\begin{vmatrix} 3 & -1 & 0 \\ 2 & 3 & 5 \\ -2 & 1 & 4 \end{vmatrix} = 3 \begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix} - (-1) \begin{vmatrix} 2 & 5 \\ -2 & 4 \end{vmatrix} = 3(12 - 5) + (8 + 10) = 39.$$

Deluppgift B

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 2 \\ 4 & 3 & 2 & -1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 3 & 2 & -1 \end{vmatrix} + (-1) \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 2 & -1 \end{vmatrix} - 4 \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 2 & 2 \end{vmatrix}$$

där

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 3 & 2 & -1 \end{vmatrix} = \dots = -12,$$
$$\begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 2 & -1 \end{vmatrix} = \dots = -2 \quad \text{och}$$
$$\begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 2 & 2 \end{vmatrix} = \dots = -2$$

så att

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 2 \\ 4 & 3 & 2 & -1 \end{vmatrix} = -12 + 2 + 8 = -2.$$

Deluppgift C

$$\det A := \begin{vmatrix} t & 1 & 2 & 3 \\ 2t & 2+t & 4-t & 7 \\ 0 & 0 & t & 2 \\ t & 1 & 2+t & 7 \end{vmatrix} = t \begin{vmatrix} t & 1 & 3 \\ 2t & 2+t & 7 \\ t & 1 & 7 \end{vmatrix} - 2 \begin{vmatrix} t & 1 & 2 \\ 2t & 2+t & 4-t \\ t & 1 & 2+t \end{vmatrix}$$

där

$$\begin{vmatrix} t & 1 & 3 \\ 2t & 2+t & 7 \\ t & 1 & 7 \end{vmatrix} = \begin{vmatrix} t & 1 & 3 \\ 2t & 2+t & 7 \\ 0 & 0 & 4 \end{vmatrix} = 4 \begin{vmatrix} t & 1 \\ 2t & 2+t \end{vmatrix} = 4(t^2 + 2t - 2t) = 4t^2$$

och

$$\begin{vmatrix} t & 1 & 2 \\ 2t & 2+t & 4-t \\ t & 1 & 2+t \end{vmatrix} = \begin{vmatrix} t & 1 & 2 \\ 2t & 2+t & 4-t \\ 0 & 0 & t \end{vmatrix} = t \begin{vmatrix} t & 1 \\ 2t & 2+t \end{vmatrix} = t(t^2 + 2t - 2t) = t^3$$

så att

$$\det A = t \cdot 4t^2 - 2 \cdot t^3 = 4t^3 - 2t^3 = 2t^3.$$