

Uppgift A 5.13 d

$$\begin{aligned}\int \frac{2x-3}{x^2+4x+13} dx &= \int \frac{2x-3}{(x+2)^2+9} dx = \\ &= \int \frac{2(x+2)-7}{(x+2)^2+9} dx = \left| \begin{array}{l} t = x+2 \\ dt = dx \end{array} \right| = \int \frac{2t-7}{t^2+9} dt = \\ &= \int \frac{2t}{t^2+9} dt - 7 \int \frac{1}{t^2+9} dt = \\ &= \ln(t^2+9) - \frac{7}{9} \int \frac{9}{t^2+9} dt = \\ &= \ln(t^2+9) - \frac{7}{9} \int \frac{1}{\frac{t^2}{9}+1} dt = \\ &= \ln(t^2+9) - \frac{7}{9} \int \frac{1}{\left(\frac{t}{3}\right)^2+1} dt = \left| \begin{array}{l} s = t/3 \\ t = 3s \\ dt = 3ds \end{array} \right| = \ln(t^2+9) - \frac{7}{3} \int \frac{1}{s^2+1} ds = \\ &= \ln(t^2+9) - \frac{7}{3} \arctan s + C = |\text{Byt ut } s| = \\ &= \ln(t^2+9) - \frac{7}{3} \arctan \frac{t}{3} + C = |\text{Byt ut } t| = \\ &= \ln(x^2+4x+13) - \frac{7}{3} \arctan \frac{x+2}{3} + C.\end{aligned}$$

Integralen av en rationell funktion är i allmänhet en summa av en logaritm och en arcustangent.