

Uppgift E4.3

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$$\begin{aligned}\frac{d}{dx} \ln(x + \sqrt{1+x^2}) &= \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x\right) = \frac{1 + \frac{x}{\sqrt{1+x^2}}}{x + \sqrt{1+x^2}} = \frac{\frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}}}{x + \sqrt{1+x^2}} = \\ &= \frac{1}{\sqrt{1+x^2}}.\end{aligned}$$

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$$\frac{d}{dx} \ln \left| \tan \frac{x}{2} \right| = \frac{1}{\tan \frac{x}{2}} \cdot \frac{1}{\cos^2 \frac{x}{2}} \cdot \frac{1}{2} = \frac{1}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{1}{\sin \left(2 \frac{x}{2}\right)} = \frac{1}{\sin x} (= \csc x).$$